Statistical and Low Temperature Physics (PHYS393)

8. Liquid Helium-3

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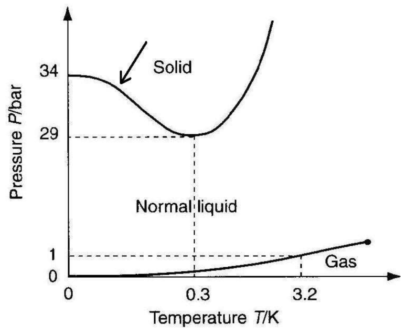
1. Phase diagram
2. Pomeranchuk effect
3. He-3 superfluid
4. Cooper pairs
5. Experimental confirmation.

Helium-3 phase diagram.

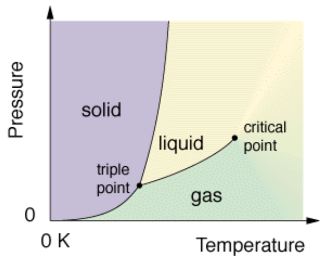
This figure shows the phase diagram for helium-3.



Guenault, Basic Superfluids (2003)

Look at the solid-liquid boundary (arrow) below 0.3 K. It has a negative slope.

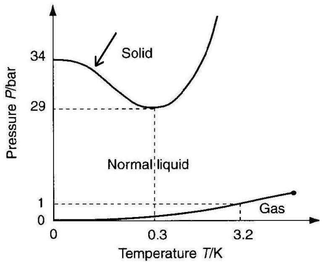
The solid-liquid boundary for most materials (except water) has a positive slope.



http://serc.carleton.edu/research_education/equilibria/phaserule.html

The reason is that higher pressure forces the atoms together. This normally helps to form the solid, so a so the liquid can freeze at a higher temperature (i.e. more easily).

For liquid helium-3 below 0.3 K, a higher pressure results in lower melting point instead.



Even though the higher pressure forces the atoms together, the liquid cannot freeze unless the temperature is reduced further.

This is very unusual. There must be a special reason for this.

Entropy.

For the atoms. the main difference between solid and liquid is the ordering.

In a normal material, the atoms would be more ordered in the solid where they are fixed, and more disordered in the liquid where they move randomly.

Therefore, we can look for clues in the entropy of solid and liquid helium-3.

Recall that the helium-3 atom is a fermion. Let us review what we know about entropy of fermions.

Liquid helium-3.

In the lectures on dilution refrigerator, we have seen that liquid helium-3 can be described as a Fermi gas.

This means that we can use the methods for electrons in metals to calculate heat capacities and entropies in liquid helium-3 - provided we assume that the helium-3 atoms acquires a different (effective) mass.

Recall that we can use the formula for heat capacity of electrons in metal:

$$C = \frac{\pi^2}{2}\frac{T}{T_F}R$$

and the relation

$$S = \int_0^T \frac{C_3(T')}{T'}dT'$$

to obtain the entropy for liquid helium-3:

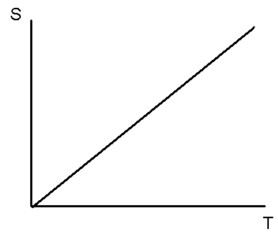
$$S = 22T \text{ J/(mol K)}.$$

Temperature variation.

The entropy for liquid helium-3 is linear in temperature:

$$S = 22T \text{ J/(mol K)}.$$

So a graph would simply look like this:

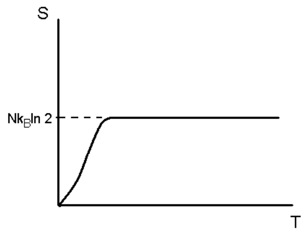


Solid helium-3.

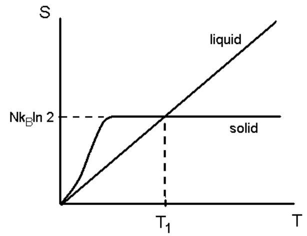
In the solid, the helium-3 atoms are fixed, so we cannot model it as a Fermi gas.

However, we can still think of them like electrons - each helium atom has a magnetic moment.

This is just what we have seen in paramagnetic salts. From the lectures on paramagnetic salts and nuclear cooling, we know that the entropy looks like this:



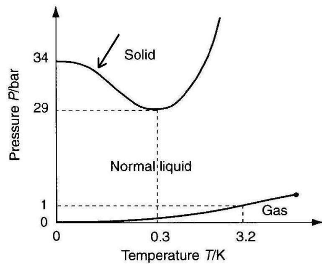
The different behaviour in entropies of the solid and liquid means that they cross over at some temperature T_1.



Above T_1, the liquid entropy is higher (more disordered) than the solid entropy, as we would normally expect.

Below T_1, the solid entropy is higher than the liquid entropy. This means that the solid is more disordered!

Consider the liquid next at a point near the arrow.

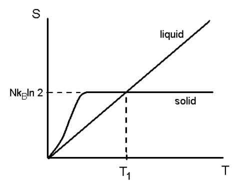


When temperature is increased, entropy increases. The only way this can happen here is to change to solid, which has higher entropy.

So the liquid *freezes* when temperature rises. This explains the negative slope below 0.3 K.

Pomeranchuk cooling.

We have seen that when liquid helium-3 becomes solid below 0.3 K, entropy increases.



In order for entropy to increase, heat must be absorbed from the surrounding (like when a normal solid melts). This gives rise to a cooling effect.

Higher pressure forces the liquid to become solid. This gives a cooling effect. This “compressional cooling” method was proposed by Issak Pomeranchuk in 1950.

Magnetic disorder.

Let us briefly review the reason why solid helium-3 can be more disordered that the liquid.

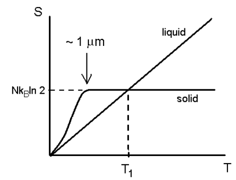
We normally think that a liquid is more disordered than a solid, because in a liquid the atoms move randomly. In measuring disorder with entropy, however, we consider the possible arrangements in energy levels, not in spatial positions.

Since liquid helium-3 is a Fermi gas, it is stacked up in energy levels in an orderly manner. At low temperatures, only a small fraction near the Fermi energy would be excited (disordered).

Magnetic disorder.

Solid helium-3 behaves like a paramagnetic solid. In this case, it is the nuclear magnetic moment that is important. (Electronic magnetic moment is zero - the moments of the 2 electrons in each atom cancel.)

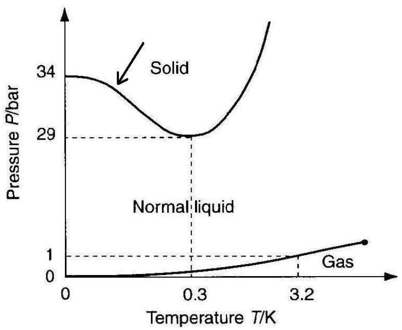
We have seen in the lectures on nuclear cooling that the ordering temperature for the nuclei is of the order of 1 μK. Above this temperature, the entropy quickly reaches the final value of $Nk_B \ln 2$ - i.e. completely disordered.



As a result, it is possible for the solid to be more disordered (in energy levels) than the liquid between 1μK and 0.3 K.

Superfluid Helium-3.

Unlike the helium-4, the phase diagram below does not show the superfluid phase.



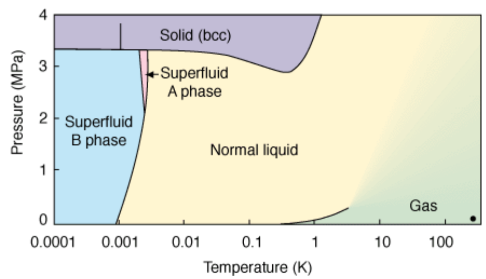
For many years, it was believed that superfluid is not possible for helium-3. It is a fermion, and Bose-Einstein condensation cannot happen.

That is, until the arrival of the BCS theory.

Updated phase diagram

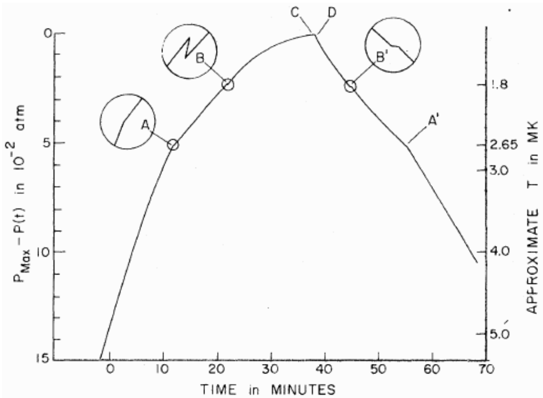
The BCS theory suggests that it may possible for helium-3 atoms to form Cooper pairs, just like electrons.

In 1972, using Pomeranchuk cooling, the superfluid of helium-3 was finally discovered by Osheroff, Richardson and Lee.



This updated phase diagram shows helium-3 superfluid below 2.5 mK.

We shall now look at some of the experimental evidence for helium-3 superfluid, starting with the discovery in 1972.



Osheroff, Richardson and Lee, Phys. Rev. Lett., vol. 28 (1972) p. 885

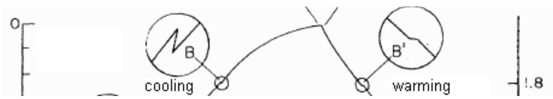
In 1972, Osheroff, Richardson and Lee cooled liquid helium-3 using the Pomeranchuk method.

They increased the pressure and recorded the falling temperature until about 1 mK. Then they release the pressure and continued recording the rising temperature.

They plotted the graph and expected to see a smooth curve. They saw 2 small kinks on the way up, and 2 tiny glitches on the way down.

After careful measurements, they concluded that these were due to the formation of 2 new phases, which the called A and B.

The most obvious sign is the tiny plateau at B' on warming.



This is just the kind or plateau we would see if we measure the temperature of melting ice.

The reason it looks different at B on cooling is explained as supercooling.

Supercooling could also happen to water, where temperature might cool slightly below 0 deg C before the ice freezes, when temperature would jump back to zero.

The meaning of the bends at A and A' are less obvious, but can also be explained in terms of phase transitions.

However, whereas B and B' are first order transition, A and A' are second order transition.

Second order transitions are phase transitions in which latent heat is zero. It is less common, but examples include:

- transition through triple point of water
- transition from para to ferromagnet
- transition from normal to superconductor

That the transition to phase A and the transition to superconductor are both second order suggests possible connection.

Soon after the discovery of the new phases, Anthony Leggett proposed that phases A and B are the result of helium-3 atoms forming Cooper pairs.

The difference is that in phase A, the spins are parallel, so that the total spin is 1. In phase B, the spins are opposite, so total spin is 0.

In both cases, the helium-3 atoms pair up to form bosons.

This means that Bose-Einstein condensation and superfluidity are then possible.

Experiments.

The suggestion that phases A and B are Cooper pairs is reasonable, but needs to be checked by experiments.

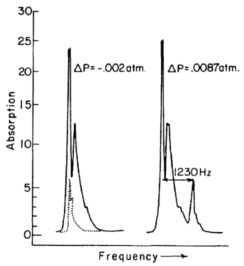
One experiment is to measure the magnetic energy levels. If phase A has spin 1, it would have magnetic levels in a field. Phase B has zero spin, so there would be no levels.

Photons from a high frequency magnetic field at the right energies could be absorbed and re-emitted, just as in atomic spectra.

This way of measuring the magnetic energy levels is called Nuclear Magnetic Resonnace (NMR).

NMR spectrum

The NMR spectrum is taken as liquid helium-3 is cooled below 2.5 mK. A new peak from phase A appeared. On further cooling to phase B, the peak disappeared.



Osheroff, et al, Physical Review Letters, vol. 29 (1972) p. 920

This observation is consistent with the theory that Phase A contains Cooper pairs with spin 1, and Phase B Cooper pairs with spin 0.

BCS theory.

Another way to verify that phase B contains Cooper pair with spin 0 is to check this relation from BCS theory:

$$2\Delta = 3.52 k_B T_c.$$

Using the phase B transition temperature T_c, the energy gap Δ can be predicted.

One experiment to measure Δ is to use ultrasound. Ultrasound is transmitted into phase B. The frequency ω is increased.

If there are indeed Cooper pairs, they will break up when the phonon energy $\hbar\omega$ reaches Δ. When this happens, the ultrasound will be strongly attenuated, since it has to give up its energy to break up the Cooper pairs.

Ultrasound.

The following shows the ratio of the energy gap Δ_{expt} measured from the ultrasound experiment, to the energy gap Δ_{BCS} predicted by the BCS theory.

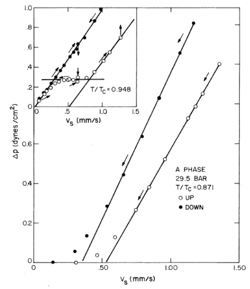
Pressure (bars)	$\frac{\Delta_{expt}}{\Delta_{BCS}}$
4.85	0.994
9.80	1.030
18.10	1.050

Movshovich, Kim, and Lee, Phys. Rev. Lett. 64, p. 431 (1990)

The ratio is quite close to 1, showing that the BCS prediction is accurate. This provides further evidence that the Cooper pairs exist.

Flow viscosity.

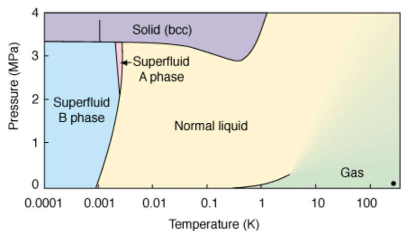
In another experiment, phase A flows through a narrow channel. The pressure difference between the two ends of the channel is measured.



Paalanen and Osheroff, Phys. Rev. Lett. 45, p. 362 (1980)

The result shows that below transition temperature, the pressure difference fall to zero. No pressure is needed to keep phase A flowing. This means it has no viscosity, confirming that it is a superfluid.

The experiments on superfluid helium-3 provide strong evidence that our understanding of helium-4 superfluid, superconductivity, and Bose-Einstein condensate is correct.



Many questions remain, such as why superfluid helium-4 is only 10% BEC, why the BCS theory cannot fully explain high T_c superconductors, and so.

Exercise 1.

Using the Fermi gas entropy for liquid helium-3

$$S = 22T \text{ J/(mol K)}$$

and the maximum magnetic entropy for the solid

$$S = N_A k_B \ln 2,$$

predict the temperature at which the slope of the solid-liquid boundary in the phase diagram changes sign. How does it compare with the measured value of 0.3 K?